DRAFT GCE A level Pure Mathematics (9MA0) – Paper 2
Pure Mathematics 2

Summer 2019 Shadow Paper student-friendly mark scheme

Please note that this DRAFT mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn't show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.

This document is intended for guidance only and may differ significantly from the mark schemes used by examiners.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question 1 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$2^y \times (2^5)^x = 2^{-3} \times 2^{-\frac{1}{2}} \Rightarrow 2^{5x+y} = 2^{-\frac{7}{2}}$	M1	This mark is given for writing all terms in the same base and applying an index law
	$5x + y = -\frac{7}{2}$	M1	This mark is given for writing an equation to link x and y
	$y = -5x - \frac{7}{2}$	A1	This mark is given for rearranging to find a correct expression of y as a function of x

Question 2 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$(\frac{1}{2} \times 2)(2 + 26 + 2 \times (7 + 9 + 13 + 22))$	M1	This mark is given for a method to use the trapezium rule as an approximation to the area under the curve
		M1	This mark is given for a correct terms used for the trapezium rule
	130 m	A1	This mark is given for a correct estimate of the length of the runway
(b)	An overestimate since the area of the five trapezia is greater than the area under the curve	B1	This mark is given for a valid explanation

Question 3 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	The formula is only valid when the angle AOB is given in radians	B1	This mark is given for a correct explanation
(b)	$\frac{36}{360} \times 2\pi \times 0.5 \times 7^2$	M1	This mark is given for a correct method to find the area of the sector
	$\frac{49\pi}{10}$ cm^2	A1	This mark is given for a correct value for the area of the sector

Question 4 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$(22\cos t)^2 + (2\sqrt{5}\sin t)^2 = 130$	M1	This mark is given for combining the two equations to show where the curve and circle meet
	$484(\cos t)^2 + 20(1 - \cos t)^2 = 130$	M1	This mark is given for forming an equation in cos t only
	$464\cos^2 t = 110$	A1	This mark is given for simplifying to find an equation in terms of cos t
	$\cos t = \frac{55}{232}$ $\Rightarrow t = 1.06\ldots, 5.22\ldots, 2.07\ldots, 4.30\ldots$	M1	This mark is given for finding a value for t. Note that there are four values for four intersections and we are looking for the value of t that gives a positive value for x and a negative value for y
	$x = 22 \times \sqrt{\frac{55}{232}} = 10.7$ $y = 4\sqrt{5} \times -\sin 5.22\ldots = -3.91$	M1	This mark is given for a method to substitute back into the original equations to find value for x and y. Note that x has to be positive and so is the positive answer, and that there are two values of t that give the correct negative y value.
	$S = (10.7, -3.91)$	A1	This mark is given for the correct coordinates of S to 3 significant figures

Question 5 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\lim_{\delta x \to 0} \sum_{x=1}^{16} \lvert x \rvert \, \delta x = \int_1^{16} \sqrt{x} \, \mathrm{d}x$	B1	This mark is given for writing the expression for a sum as an integral. The modulus sings are dropped since x is not negative within the domain of the integral bounds.
	$\left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^{16} = \frac{2}{3} \times 16^{\frac{3}{2}} - \frac{2}{3}$	M1	This mark is given for a method to evaluate the integral
	$= 42$	A1	This mark is given for a correct evaluation of the integral

Question 6 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$gg(0) = g(10)$	M1	This mark is given for a method to find $gg(0)$
	$g(10) = 22$	A1	This mark is given for a correct value for $gg(0)$
(b)	$(x-3)^2 + 1 > 9$ $(x-3) > \sqrt{8}$ $x > 3 + 2\sqrt{2},\ x < 3 - 2\sqrt{2}$	M1	This mark is given for a method to solve $g(x) > 9$ when $x \leq 3$
	$x < 3 - 2\sqrt{2}$	A1	This mark is given for the correct answer only
	$3x - 8 > 9$ $x > \frac{17}{3}$	M1	This mark is given for a solving $g(x) < 9$ when $x < 0$
	$x < 3 - 2\sqrt{2}$ and $x > \frac{17}{3}$	A1	This mark is given for a correct range of values of x for which $g(x) < 9$ stated
(c)	h^{-1} exists since h is a one-to-one function; g^{-1} does not exists since g is a many-to-one function	B1	This mark is given for a valid explanation
(d)	$h^{-1}(x) = 6 - \sqrt{(x-6)}$	B1	This mark is given for finding an expression for $h^{-1}(x)$. Note the negative root is taken since the domain of $h(x)$ is $x \leq 6$, so $h(x) \leq 6$ also
	$6 - \sqrt{(x-6)} = -2$ $\sqrt{(x-6)} = 8$	M1	This mark is given for a method to rearrange to find a value for x
	$x = 70$	A1	This mark is given for a correct value of x

Question 7 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = C + Kx$, where C and K are constants	B1	This mark is given for stating a correct general equation
(b)	$200 = 650 \times 5 - (C + 650k)$ $-80 = 230 \times 5 - (C + 230k)$	M1	This mark is given for modelling the profit on the two days when pies are sold for £5
	$C + 650K = 3050$ $C + 230K = 1230$	M1	This mark is given for forming a pair of simultaneous equations to find values for C and K
	$420K = 1820 \Rightarrow K = \frac{13}{3}$ $C = 1230 - (230 \times \frac{13}{3}) = \frac{700}{3}$ Thus $y = \frac{13}{3}x + \frac{700}{3}$	A1	This mark is given for finding the values of C and K to find an equation in y
(c)	The gradient represents the cost of making each extra pie in £s	B1	This mark is given for a valid interpretation of the significance of the gradient
(d)	For n pies $2n - (428 + 0.84n) > 0$	M1	This mark is given for a method to find the number of pies to be made
	$1.16n - 428 > 0$ $n - \frac{428}{1.16} > 0$ $n = 369$ pies	A1	This mark is given for correctly finding the number of pies to be made

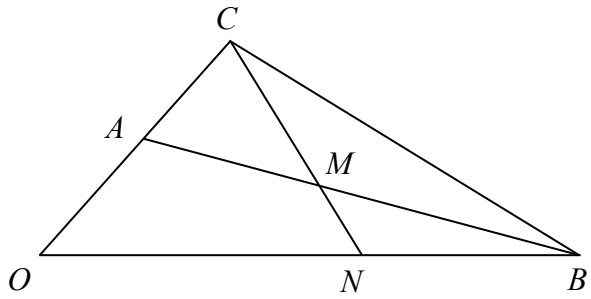
Question 8 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(i)	$\sum_{k=3}^{\infty} 6\times\left(\frac{1}{2}\right)^k$ $= \sum_{k=1}^{\infty} 6\times\left(\frac{1}{2}\right)^k - 6\left(\frac{1}{2}+\frac{1}{4}\right)$ $= \sum_{k=1}^{\infty} 6\times\left(\frac{1}{2}\right)^k - \frac{9}{2}$	M1	This mark is given for a method to find the sum to infinity of a GP
	$= \frac{3}{1-\frac{1}{2}} - \frac{9}{2}$	M1	This mark is given for a method to use a correct sum formula with a correct first term
	$= \frac{3}{2}$	A1	This mark is given for a correct value for the sum
(ii)	$\sum_{n=0}^{125} \log_4\left(\frac{k+3}{k+2}\right)$ $= \log_4 \frac{3}{2} + \log_4 \frac{4}{3} + \ldots + \log_4 \frac{127}{126} + \log_4 \frac{128}{127}$	M1	This mark is given for writing out at least four terms of the sum, including the first two and the last two
	$= \log_4 \frac{3\times4\times\ldots\times126\times127\times128}{2\times3\times4\times\ldots\times126\times127}$ $= \log_4 \frac{128}{2}$	M1	This mark is given for using the rules of logs and cancelling terms
	$= 3$	A1	This mark is given for a full proof to show the expression is equal to 3 as required

Question 9 (Total 9 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	If $d = kV^n$, then $\log_{10} d = \log_{10} k + n \log_{10} V$	M1	This mark is given for finding an appropriate log relationship between d, k, V and n
	Plotting $\log_{10} d$ against $\log_{10} V$ will result in a straight line with gradient n and intercept $\log_{10} k$	A1	This mark is given for an explanation of why the second graph shows that $d = kV^n$
	$\log_{10} k = -1.77$ $k = 10^{-1.77} = 0.01698\ldots \approx 0.017$	A1	This mark is for showing fully that $k \approx 0.017$
(b)	$\log_{10} d = -2$, $\log_{10} V = 0$ $-2 = \log_{10} k$ $k = 0.01$	B1	This mark is given for using Figure 6 to obtain a value for k
	When $V = 24$, $d = 16$ $\log_{10} 16 = -2 + n \log_{10} 24$	M1	This mark is given for substituting in the formula as a method to find the value of n
	$n = \dfrac{\log_{10} 16 + 2}{\log_{10} 24}$	M1	This mark is given for a correct expression for n
	$n = 2.32$ to 3 significant figures $d = 0.001 \times V^{2.32}$	A1	This mark is given for finding a correct value of n to 3 significant figures and writing a complete equation for the model
(c)	$\dfrac{44}{3600} \times 0.8 \times 1000 = 9.77\ldots$ m	M1	This mark is given for a method to find the distance, in metres, covered in the reaction time of 0.8 seconds
	$d = 0.017 \times 44^{2.32} = 65.347\ldots$ m	M1	This mark is given for a method to use the formula to find the stopping distance
	9.77 m + 65.35 m = 75.12 m Sean will **not** be able to stop before reaching the puddle	A1	This mark is given for finding a correct value of the total stopping distance and giving a valid conclusion

Question 10 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
			
(a)	$\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{2}{3}\overrightarrow{AB}$	M1	This mark is given for a method to find an expression for $\overrightarrow{CM}$
	$\overrightarrow{CM} = -\mathbf{a} + \frac{2}{3}(\mathbf{b} - \mathbf{a}) = -\frac{5}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$	A1	This mark is given for a correct expression for $\overrightarrow{CM}$ in terms of **a** and **b**
(b)	$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} = \overrightarrow{OC} + k\overrightarrow{CM}$	M1	This mark is given for a method to find an expression for $\overrightarrow{ON}$
	$\overrightarrow{ON} = 2\mathbf{a} + k\left(-\frac{5}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}\right)$ $= \left(2 - \frac{5}{3}k\right)\mathbf{a} + \left(\frac{2}{3}k\right)\mathbf{b}$	A1	This mark is given for a correct expression for $\overrightarrow{ON}$ in terms of **a** and **b**
(c)	$\left(2 - \frac{5}{3}k\right) = 0$ so $k = \frac{6}{5}$	M1	This mark is given for deducing that the coefficient of **a** = 0 and finding a value for k
	$\overrightarrow{ON} = 0 \times \mathbf{a} + \left(\frac{2}{3} \times \frac{6}{5}\right)\mathbf{b} = \frac{4}{5}\mathbf{b}$ Hence $ON:NB = \frac{4}{5}:\frac{1}{5} = 4:1$	A1	This mark is given for finding $\overrightarrow{ON}$ and giving a valid conclusion

Question 11 (Total 11 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = x^x \Rightarrow \ln y = x \ln x$	M1	This mark is for a method to find the x-coordinate of the turning point of C by taking logarithms
	$\ln y = x \ln x \Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln x + 1$	M1	This mark is given for a method using implicit differentiation
		A1	This mark is given for a correct expression for $\frac{1}{y}\frac{dy}{dx}$
	Setting $\frac{dy}{dx} = 0$, $\ln x + 1 = 0$	M1	This mark is given for a method for finding the turning point of C by setting $\frac{dy}{dx} = 0$
	$x = e^{-1}$	A1	This mark is given for correctly finding a value for the x-coordinate of the turning point of C
(b)	$1.5^{1.5} = 1.837\ldots$, $1.6^{1.6} = 2.121\ldots$	M1	This mark is given for substituting 1.5 and 1.6 into $y = x^x$
	The curve C contains the points (1.5, 1.8) and (1.6, 2.1). At P, $y = 2$ Since C is continuous, $1.5 < \alpha < 1.6$	A1	This mark is given for a valid explanation that C contains the points (1.5, 1.8) and (1.6, 2.1) and is continuous
(c)	$x_1 = 1.5$ $x_2 = 2 \times 1.5^{-0.5} = 1.633$	M1	This mark is given for finding a correct value for x_2
	$x_3 = 2 \times 1.633^{-0.633} = 1.466$ $x_4 = 2 \times 1.466^{-0.466} = 1.673$	A1	This mark is given for finding a correct value for x_4
(d)	For example: x_n oscillates is periodic is non-convergent	B1	This mark is given for a valid statement about the long-term behaviour of x_n
	between 1 and 2	B1	This mark is given for stating that the behaviour is between 1 and 2

Question 12 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{\cos 3\theta}{3\sin\theta} + \frac{\sin 3\theta}{3\cos\theta} = \frac{\cos 3\theta\cos\theta + \sin 3\theta\sin\theta}{3\sin\theta\cos\theta}$	M1	This mark is given for a method to form a single fraction
	$= \frac{\cos(3\theta - \theta)}{3\sin\theta\cos\theta}$	M1	This mark is given for a method to use a compound angle formula on the numerator
	$= \frac{\cos 2\theta}{\frac{3}{2}\sin 2\theta}$	M1	This mark is given for a method to use a compound angle formula on the denominator
	$= \frac{3}{2}\cot 2\theta$	A1	This mark is given for a fully correct proof to show the answer required
(b)	$\frac{3}{2}\cot 2\theta = 1$ $\tan 2\theta = \frac{3}{2}$	M1	This mark is given for deducing that the value of $\tan 2\theta$
	$\theta = \frac{3}{2}\arctan\frac{3}{2}$	M1	This mark is given for finding an expression for a solution for θ
	$\theta = 28.2°, 118.2°$	A1	This mark is given for finding two correct values for θ

Question 13 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$5.4 = \pi r^2 h + \frac{2}{3}\pi r^3$	B1	This mark is given for a method to find the volume of the cylinder and the semi-hemisphere
	$A = 3\pi r^2 + 2\pi\left(\frac{5.4 - \frac{2}{3}\pi r^3}{\pi r}\right)$	M1	This mark is given for a method to find the surface area of the tank
		A1	This mark is given for finding an expression for the surface area of the tank
	$A = 3\pi r^2 + \frac{10.8}{r} - \frac{4\pi r^2}{3}$ $= \frac{10.8}{r} + \frac{5\pi r^2}{3}$	A1	This mark is given for the correct answer only
(b)	$A = \frac{10.8}{r} + \frac{5\pi r^2}{3}$	M1	This mark is given for a method to differentiate to find r
	$\Rightarrow \frac{\mathrm{d}A}{\mathrm{d}r} = -\frac{10.8}{r^2} + \frac{10\pi r}{3}$	A1	This mark is given for accurately differentiating to find r
	When $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$, $-\frac{10.8}{r^2} + \frac{10\pi r}{3} = 0$ $r^3 = 3.24\pi$	M1	This mark is given for a method to set $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ to find a value for r
	$r = 2.17$	A1	This mark is given for finding the radius for which the surface area is a minimum
(c)	$A = \frac{10.8}{0.998} + \frac{5\pi(0.998)^2}{3}$	M1	This mark is given for a method to substitute a value for r
	$A = 29.63$ m^2 $A = 30$ m^2	A1	This mark is given for correctly finding the minimum surface area of the tank (to the nearest integer)

Question 14 (Total 15 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\mathrm{d}h = -2(6-u)\,\mathrm{d}u$	B1	This mark is given for finding an expression for dh
	$\int \frac{\mathrm{d}h}{6-\sqrt{h}} = \int \frac{-2(6-u)\,\mathrm{d}u}{6-\sqrt{h}}$	M1	This mark is given for substituting $u = 6 - \sqrt{h}$ into the integral
	$= \int -\frac{12}{u} + 2\,\mathrm{d}u$	M1	This mark is given for a method to find a simplified version of the integral
	$-12\ln u + 2u + c$ $= -12\ln(6-\sqrt{h}) + 2(6-\sqrt{h}) + c$	M1	This mark is given for integrating with respect to u to produce an expression in terms of h
		A1	This mark is given for a correct expression for the integral
	$= -12\ln(6-\sqrt{h}) - 2\sqrt{h} + k$	A1	This mark is given for a full proof to arrive at the answer as shown (appreciating that $k = c + 12$)
(b)	$\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \Rightarrow 6 - \sqrt{h} = 0$	M1	This mark is given for a setting $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$
	$0 < h < 36$	A1	This mark is given for deducing the range of the heights of the trees for this model
(c)	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4-\sqrt{h})}{20} \Rightarrow \frac{\mathrm{d}h}{(4-\sqrt{h})} = \frac{t^{0.25}\mathrm{d}t}{20}$	B1	This mark is given for separating the variables
	$-12\ln(6-\sqrt{h}) - 2\sqrt{h} + k = \frac{t^{1.25}}{25}$	M1	This mark is given for a method to integrate both sides of the equation
		A1	This mark is given for integrating both sides of the equation correctly
	When $t = 0$ and $h = 1$, $-12\ln 5 - 2 + k = 0$ $k = 2 + 12\ln 5$	M1	This mark is given for substituting values of $t = 0$ and $h = 1$ to find a value for k
	When $h = 15$, $-12\ln(6-\sqrt{15}) - 2\sqrt{15} + 2 + 12\ln 5 = \frac{t^{1.25}}{25}$	M1	This mark is given for a method to find a value for t by substituting $h = 15$ into the equation
	$t^{1.25} = 112.7661\ldots \Rightarrow t = \sqrt[1.25]{112.7661}$	M1	This mark is given for simplifying to find an expression for t
	$t = 43.83$ years	A1	This mark is given for correctly finding the time the tree would take to reach a

Part	Working or answer an examiner might expect to see	Mark	Notes
			height of 15 metres